Neutrinos as probes for quantum gravity

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Abstract

In this article, we will explore how neutrinos interact with gravity, with a particular focus on how these particles can be used as probes to investigate the presumed quantized structure of spacetime. Indeed, any modification to the formulation of gravitational interaction is expected to impact particle physics. We will therefore study how neutrinos can be used to test both universal and non- universal perturbations arising from the phenomenology of quantum gravity. We will analyze how, in a scenario of universal modification, the flight time of particles acquires a dependence on their energy. We will also examine how the phenomenon of oscillations is affected by the introduction of corrections dependent on the particle eigenstate considered.

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1. INTRODUCTION

Studying the Planck-scale structure of spacetime is essential for formulating a complete theory of quantum gravity (QG). Various models predict different phenomenological scenarios in this context. All these models share the prediction that QG residual effects manifest as small perturbations that modify the kinematics of particles, used as probes for the spacetime Planck scale. Moreover, in this research field, the need to test the universality of the predicted QG corrections arises. The modifications to kinematics can be universal for every particle species; otherwise, the corrections may depend on the type of probe considered. The search for nonuniversal corrections can serve as a test for violations of the Weak Equivalence Principle (WEP), induced by the anticipated perturbations. The ideal framework for conducting research on QG is that of astroparticles, due to their high energy and long propagation paths, which allow for the accumulation of the predicted tiny perturbations. Neutrinos are the ideal candidates for testing these corrections, since they enable studies across different combinations of energies and baselines. Moreover, due to their extremely weak interactions, these particles can propagate over cosmic distances, unlike other cosmic messengers, whose propagation is suppressed [1, 2]. Additionally, since these particles interact weakly, they are not deflected during propagation, which offers advantages in pinpointing the sources.

1.1. Background

In this work, we present a review of some QG theories that predict phenomenological testable effects. In some of these models such as the Doubly Special Relativity (DSR) [3, 4, 5] and the Homogeneously Modified Special Relativity (HMSR) [6], covariance and the kinematic symmetry group are amended. As a consequence, the kinematics is modified and the main QG effects pertain to particle propagation. Under the assumption of universal QG corrections, the first detectable effect predicted by these models is related to the differences in the time of flight of particles with various energies. QG may affect the in-vacuum dispersion relations, leading to an energy dependence of the particle velocity. This effect can be detected in the energy spectrum of GRB and Supernova emitted neutrinos. Including the nonuniversality of the perturbations, neutrinos can be used to test the validity of the WEP [7, 8]. In fact, under the hypothesis of nonuniversal corrections, QG can introduce differences in the expected neutrino oscillation pattern. The introduction of mass eigenstate-dependent QG perturbations may alter the oscillation probability. In addition, CPT violations can, for instance, be studied in the context of the Standard Model Extension (SME) [10, 11]. The CPT symmetry is strictly related to the Lorentz covariance. Indeed, the violation of the CPT symmetry implies the explicit Lorentz Invariance Violation (LIV). In the SME scenario, the residual effects of QG manifest as an explicit violation of Lorentz invariance, with a consequent modification of the kinematics and the interaction of particles.

2. MAIN CONTENT

As the first step, we will illustrate some different QG models, within the context of which the phenomenological predictions have been obtained. We will particularly focus on DSR [3, 4, 5] and HMSR [6] that predict modifications to the kinematic symmetry group. We will also discuss how nonuniversal corrections can be incorporated into these models. Then, we will consider LIV and then SME [10, 11], to illustrate how the CPT-odd extension of the Standard Model in neutrino sector has a formulation similar to Non Standard Interactions (NSI). Finally, we will analyze the phenomenology introduced in neutrino physics, illustrating how this particle can be used as probes for QG different models

2.1. Theoretical Framework

The theoretical motivation underlying some of the most studied QG models is that real physics takes place in the phase space, where the interactions of different particles are described. The spacetime is constructed through measurements and observations. The spacetime is therefore a momentum space projection obtained locally by every observer. In this context, we can set DSR [3, 4, 5] and HMSR [6] theories.

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2.1.1. DSR theories

In this class of QG theoretical models [3, 4], the geometry is defined in the momentum space and is determined by the modified composition rules of momenta:

$$p_1 \otimes p_2 = p_1 + p_2 + f(p_1, p_2) \tag{1}$$

where f is an opportune function of the momenta p_1 and p_2. The space connection is obtained as follows

$$\frac{\partial}{\partial p_{1\mu}} \frac{\partial}{\partial p_{2\nu}} (p_1 \otimes p_2)_\tau = \Gamma^{\mu\nu}_\tau \tag{2}$$

The momentum space curvature vanishes, but the geometry is nontrivial since the connection's origin is nonmetric. The underlying symmetry group is the κ-Poincarè´s one, associated with a Hopf algebra structure. Lorentz invariance is amended and promoted to diffeomorphism invariance. The classical formulation of Special Relativity can be obtained as the low-energy limit of the model. In the context of DSR theories, nonuniversality can be introduced by considering the Hopf algebra formulation. Different algebras, related to different corrections, can be connected by defining a projection onto a common support algebra [7, 8].

2.1.2. HMSR theory

In this model [6], QG is associated with a modification of the dispersion relations. The modified dispersion relations (MDR) are defined using homogeneous functions to guarantee their geometric origin in the context of Finsler geometry:

$$E_i^2 - |\vec{p}_i|^2 (1 - f_i(\frac{|\vec{p}_i|}{E_i})) \tag{3}$$

Lorentz invariance is modified, that is the Poincarè group is altered to preserve the MDR structure. The perturbations are conceived as dependent on the particle species. Each particle probes a spacetime whose geometry acquires an explicit dependence on its energy. The interaction of different particle species can be described using a support space, on which the different spacetimes are projected. In this context, a minimal extension of the Standard Model of particle physics can be obtained, preserving the usual SU(3) × SU(2) × U(1) gauge symmetry. The high-energy limit of this model is compatible with the original formulation of the Coleman-Glashow modified relativity [9].

2.1.3. SME

In string theory, the interaction can lead to explicit LIV by introducing vacuum expectation values different from zero for some string operators. The SME is formulated following this idea, including all possible operators that extend the Standard Model (SM) [10]. In this context, operators can be introduced which are either renormalizable or not, and preserve or violate CPT symmetry [11]. The extension of the SM preserves the usual gauge symmetry, and no exotic particles are introduced or required. In this case, Lorentz symmetry is simply violated and not amended, meaning spacetime isotropy is no longer guaranteed. For instance, the LIV terms in the Lagrangian formulation can be constructed coupling the usual terms with fixed background ad hoc introduced objects. The first resulting effect is the introduction of preferred directions in spacetime. As a consequence, for every experiment, the need of introducing a privileged reference frame arises and a sidereal and annual variation in the results is expected.

2.2. Phenomenology in the neutrino sector

Neutrinos emerge as ideal candidates for multimessenger astroparticle research [5, 7, 8]. These particles interact very weakly, making the Universe transparent to their propagation. Moreover, neutrinos detected on Earth span a wide range of energies and propagation lengths, furnishing some advantages for the search of the presumed QG perturbations. Besides, these particles offer the advantage of being able to point back to their sources, as they interact weakly and are minimally disturbed during propagation

2.2.1. Neutrinos and classical gravity

The interaction between neutrinos and gravity is also predicted in the classical General Relativity framework [7]. Here, we consider how neutrinos can be affected by the presence of a gravitational field in the classical context. As a result, the oscillation pattern is modified. The oscillation probability is defined as follows :

$$P_{(v_\alpha \to v_\beta)} = | \langle v_\beta | v_\alpha(L) \rangle | \tag{4}$$

where the evolution of an α flavor state is projected on a state of flavor β. The evolution after a propagation length L is computed using the Schrodinger equation as follows:

$$|v_\alpha(L)| = U_{\alpha k} e^{-i \int_{r_A}^{r_B} (\eta_{\mu\nu} + \frac{1}{2} h_{\mu\nu}) p_k^\mu dx^\nu |v_\alpha\rangle} \tag{5}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ is the perturbation term. The General Relativity (GR) modified phase, ruling the oscillation between j and k mass eigenstates, is obtained summing the standard phase with a gravity perturbation term:

$$\phi_{jk} = \phi_{jk}^0 + \phi_{jk}^{GR} \tag{6}$$

For instance, considering the Schwarzschild spacetime, the perturbation term acquires the following form :

$$\phi_{jk}^{GR} = \frac{\Delta m_{jk}^2}{2E} \left[\frac{G \cdot M}{r_B} - \frac{G \cdot M}{L} \log[\frac{r_B}{r_A}] \right] \tag{7}$$

where $\Delta m_{jk}^2 = m_j^2 - m_k^2$ is the usual difference of the squared mass eigenstates. r_A and r_B are the radial distances of the initial propagation point A and the final one B from the reference frame origin. M is the mass generating the Schwarzschild spacetime, and G is the gravitational constant. The classical gravity perturbations can modify the oscillation pattern and must be disentangled from the modification induced by the supposed different QG scenarios.

2.2.2. Modified dispersion relations

The main phenomenological effect introduced by QG pertains to the modification of the dispersion relations [5, 7, 8, 12]. It is not necessary to request nonuniversal modifications to introduce this effect. The kinematics of the free particle acquires an explicit dependence on the energy of the particle itself. For example, in the case of DSR models, the MDR becomes

$$E^2 - e^{2\lambda p_0} |\vec{p}|^2 \approx m^2 \tag{8}$$

and introducing the Planck mass m_{pl} as a suppression factor, the MDR can be written as follows :

$$E^2 - \frac{\delta}{m_{pl}^2} E p^2 = m^2 \hookrightarrow E = \sqrt{p^2 [1 + \frac{\delta}{m_{pl}^2} p] + m^2} \tag{9}$$

where $\lambda = \frac{\delta}{m_{pl}}$ is the constant encoding the nontrivial-trivial geometric structure of the momentum space. Applying the Hamilton's equation to the eq. (9), the particle velocity that depends on the particle energy can be computed:

$$v(E) = \frac{\partial E}{\partial p} = \frac{p + \frac{3}{2}\frac{p^2}{m_{pl}^2}}{\sqrt{p^2[1 + \frac{\delta}{m_{pl}^2}p] + m^2}} \tag{10}$$

As a result, the positive values of the correction factor δ imply particles that can propagate faster than light, whereas a negative correction is compatible with a subluminal maximum speed of propagation. In this work the positive correction is ruled out and not considered, since only subluminal particles are examined.

2.2.3. Time of flight

The dependence of propagation velocity on energy implies that particles can accumulate a time delay depending on their energy [5, 7, 8, 12, 13]. As a consequence, one of the main channels for detecting QG signatures involves measuring the time of flight of astroparticles, with neutrino as the ideal probe. In the DSR framework the contribution to the flight time as a function of the energy can be computed:

$$\Delta = \frac{|\delta|}{M_{pl}} E \int_0^Z \frac{1+\zeta}{H_0\sqrt{\Omega_\Lambda + \Omega_m(1+\zeta)}} \tag{11}$$

where $H_0, \Omega_\Lambda \Omega_m$ denote the Hubble, the cosmological constant, and the matter fraction, respectively. Z is the redshift parameter associated with the astroparticle source, and δ is the proportional coefficient encoding the QG geometry structure. In the following table, we report the computation of some time delay values as a function of the neutrino energy, computed for a QG perturbation constant $|\delta|$ = 1 for neutrinos accelerated at a distance given by the redshift parameter Z = 1:

$E_v(TeV)$	$\Delta t(s)$
1	0.5×10^2
10	0.5×10^3
50	2.5×10^4

The time delay is linearly proportional to the particle energy. This dependence of the time of flight can be detected in the energy spectrum of:

1. GRB candidate accelerated neutrinos
2. Supernova emitted neutrinos

To quantify the time delay, the source distance and neutrino beam energy are chosen to maximize the visibility of this effect. For a propagation path corresponding to a redshift parameter $Z = 1$, the vast distance of approximately 10^{10}light years results in a time delay accumulation of about 10^3 -10^4 seconds, which can be detected in the presence of QG corrections using high- energy neutrino fluxes on the order of tens up to hundreds of TeVs [7, 8, 12, 13, 14].

2.2.4. Neutrino oscillations

In the context of DSR, HMSR, and SME models, nonuniversal QG perturbations can be predicted. Introducing QG modifications that depend on the mass eigenstates can alter the oscillation phenomenon [7, 8, 15, 16, 17, 18]. The modified phase governing the oscillation pattern can be computed, for instance, in the context of the DSR theory :

$$\phi_{jk} = \left(\frac{\Delta m_{jk}^2}{2E} - \delta_{jk}E^2\right)L \tag{12}$$

while for the HMSR model:

$$\phi_{jk} = \left(\frac{\Delta m_{jk}^2}{2E} - \delta_{jk}E\right)L \tag{13}$$

where $\delta_{jk} = \delta_j - \delta_k$ is the difference of the QG perturbation parameters related to the masse eigenstates j and k respectively.Using the modified phase the integrated probability can be computed:

$$P_{(v_\mu \to v_e)} = \frac{\int_{E_{min}}^{E_{max}} \phi_v(E) P_(v_\mu \to v_e) E dE}{\int_{E_{min}}^{E_{max}} (E) dE} \tag{14}$$

This effect implies a modification of the expected events for different flavor neutrino beams, depending on the QG perturbation magnitude, and can be detected in the atmospheric sector. Indeed, atmospheric neutrinos have a long baseline and high energy, which allow for the accumulation of the predicted QG corrections.Calculating the integrated probability in an energy range of interest for the atmospheric sector, it is possible to pose constraints on the QG predicted perturbations. For instance, using the energy range from 300 MeV to 5 GeV, it is possible to consider as perturbation coefficients $\delta_{jk} \leq 10^{-23}$ in the context of HMSR theory and $\delta_{jk} \leq 10^{-32}$ in the context of DSR model.

2.2.5. CPT violation

In the SME, also the search for CPT violation can be set, investigating CPT-odd LIV perturbations [10, 18, 19]. In the Hamiltonian picture, the total operator ruling the neutrino propagation can be written as follows:

$$H_{SME} = H_0 + H^{LIV}_{CPT-even} + H^{LIV}_{CPT-odd} \tag{15}$$

The CPT-odd Hamiltonian has the same form of the NSI effective Hamiltonian:

$$H^{LIV}_{CPT-odd} = 2\sqrt{2}G_F \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{e\tau} \\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix} \tag{16}$$

The perturbations induced by supposed NSI can mimic the effect of CPT-odd QG Lorentz-violating contributions. From this observation emerges the necessity of complementary studies to disentangle the different contributions caused by QG from other beyond the Standard Model scenarios. Disentangling these contributions is crucial but challenging within a single experiment. A viable approach involves complementary studies using data from different detectors, such as comparing day/night asymmetry in neutrino interactions with NSI effects in the solar sector, while considering LIV constraints [7].

3. CONCLUSION

Some phenomenological models of QG such a DSR, HMSR, and SME, offer the possibility of being tested in the astroparticle sector. For instance, in the context of these models, both universal and nonuniversal perturbations can be searched for testing QG and potential violations of the WEP. Neutrinos can serve as ideal probes for Planck-scale physics in a multimessenger approach within the context of astroparticle physics. The neutrino sector is particularly suited for testing the presumed QG perturbations in dispersion relations, searching for modifications to the time of flight, and the resulting spectrum of candidate GRB and Supernova neutrinos. Moreover, the atmospheric sector provides an opportunity to test the universality of QG modifications by studying the oscillation pattern. Finally, in this physical sector, CPT symmetry can be tested by searching for modifications to neutrino-matter interactions, such as in the day-night asymmetry of the solar neutrino flux. Challenges remain in distinguishing QG-induced perturbations from other causes, such as NSI and classical gravitational interaction, as foreseen in General Relativity. Complementary analysis strategies to compare the results of various experiments seem necessary to achieve the goal of distinguishing the different contributions. This objective can be obtained through complementary studies that incorporate data from different detectors. For instance, comparing the results of day/night asymmetry in neutrino matter interactions with those from NSI in the solar sector, while also considering the constraints imposed on QG, can provide valuable insights

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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